ISSN 2229-5518

Simulation of Fixed Bed Heat Regenerator using Wavelets

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Abstract- Fixed bed regenerators are the heat exchangers used for transferring heatfrom hot fluid to cold fluid using solids as intermittent storage medium. The governing equations are solved by Finite Difference Method (FDM) and Wavelet method. Temperature profiles and breakthrough curves for fixed bed regenerator are drawn for both the methods. In this paper it is observed that the results obtained by Wavelet based method is more accurate and takes less computational time as compare to Finite Difference Method.

Index Terms- Fixed bed heat regenerator, Wavelet Transformation, Finite Difference Method, Grid Adaptation, Sparse Point Representation, Breakthrough Curve.

1 Introduction

The mathematical models describing diffusion processes can usually be formulated in the terms of coupled parabolic partial differential equations (PDE's).The problem is to find the efficient numerical approximation methods for the coupled transient PDE's. Solution of the models to determine the moving profiles is mostly achieved by restricting this to a zone bounded by moving fronts. Various solution methods for tracking the motion of such moving fronts have been reviewed by **Crank** and **Finlayson**.

These methods require large computation times. In comparison to these methods, orthogonal collocation is more efficient solution procedure for moving boundary problems faced in mass and heat transfer processes. In such cases to maintain an effective discretization of the spatial variable, it is necessary to use moving boundary finite elements or an adaptive technique such that the steep gradients can be tracked. The adaptive criteria are generally derived from the solutions itself.In the cases where the PDE's solution has a regular behavior, necessarily any implementation of the "traditional" numerical methods (e.g. finite differences, collocation) can be applied successfully in its resolution.

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However, singularities and steep changes often spring up in many phenomena, like concentration and temperature fronts in fixed-bed heat regenerator.Such sharp transitions in an otherwise "smooth" solution are typically moving with time along a spatial coordinate.These changes demands for the use of non-uniform grids or moving elements, that dynamically adapt to the changes in the solution. Such strategies mostly rely upon the knowledge of the solution itself. It is quite difficult to define how the solution should be precisely computed at each new added point.There is still a need for an efficient and fully adaptive method for solving this kind of problem. That is where wavelets play a vital role. A wavelet basis representation originates a set of wavelet coefficients, structured over different levels of resolution. Each coefficient is associated to a resolution level (frequency) and a point in the time (or space) domain. In case of a moving steep front, using the wavelet transform one can track its position and the local resolution of the grid increased by adding higher resolution wavelets. Similarly, the resolution level can decreased appropriately in smoother regions by avoiding an unnecessarily dense grid.

2 Wavelet transformation

Wavelets are functions generated from one single function called the mother wavelet by the simple operations of scale and translation,

$$\psi_{a,b} = a^{-1/2}\psi\left(\frac{x-b}{a}\right)$$

Where,

a- scale parameter
b- translation parameter

By using scale factor of 2^{-n} and translation factor of $k2^{-n}$, a frame corresponding to resolution level n is obtained:

$$\psi_{n,k}(x) = 2^{n/2}\psi(2^n x - k)$$

By varying k, the function is shifted on the x-axis and by varying the value of n the amplitude of the function is varied.
In the basic work of Daubechies, a family of compactly supported orthonormal wavelets is constructed. An compactly supported function have non-zero value only in a finite interval else zero everywhere. The smaller the interval, the greater is the spatial localization of the function. The Daubechies family includes elements from highly localized to highly smooth functions. Each wavelet family is governed by a set of L (an even integer) coefficients p(k) : k= 0, 1,. . ., L-1 through the two-scale relation:

$$\phi(x) = \sum_{k=0}^{L-1} p(k)\phi(2x-k)$$

Based on scaling function ϕ(x), a basis can be obtained by scaling and translating a single function ψ(x) (mother wavelet), which is defined as

$$\psi(x) = \sum_{k-2-L}^{l} q(k)\phi(2x-k)$$

Where, p(k) and q(k) are filter coefficients. Therefore, any function can be approximated at a higher resolution (n+1) from the approximation at a lower resolution (n). Consequently, f(x) is represented at resolution level n as:

$$\sum_k a_{n,k}\phi_{n,k}(x) = \sum_k a_{n-1,k}\phi_{n-1,k}(x) + \sum_k b_{n-1,k}\psi_{n-1,k}(x)$$

2.1 Sparse Point Representation

An attribute of the basis is the one-to-one correspondence between point values and wavelet coefficients. The interpolating subdivision scheme generates corresponding function values on a fine grid from the given values on a coarse grid. For odd-numbered grid points at each level, the difference between the known function value and the interpolated functionvalue from the coarser grid can be calculated. These differences are called as wavelet coefficients(d_k^j).And all the wavelet coefficients will remove whose magnitude is smaller than threshold value ε.

$$d_k^j = u(x_{2k+1}^{\ j+1}) - I_j\ u(x_{2k+1}^{\ j+1})$$

3 Simulation of Fixed Bed Heat Regenerator

Heat exchange equipments are used for transfer of heat between two fluids. The conventional way of carrying out heat exchange between two streams are tubular heat exchangers, in which the two streams are separated by heat conducting tubes. Heat transfer in a fixed-bed of solids (Regenerators) can be either a batch or continuous process. The heat exchange between cold and hot fluid carried out in a fixed bed is a two-step operation, as shown in the fig 1. Bed 1 and 2 contains hot and cold solids respectively. In the first step, hot air is passed through bed 2 and cold air through bed 1. Hot air gives up its heat to cold solids present in bed 2 while the solids release the stored heat to the cold air. In the next step, at a predetermined time the streams are switched so that cold air now passes through bed 1 and hot air through bed 2 respectively.

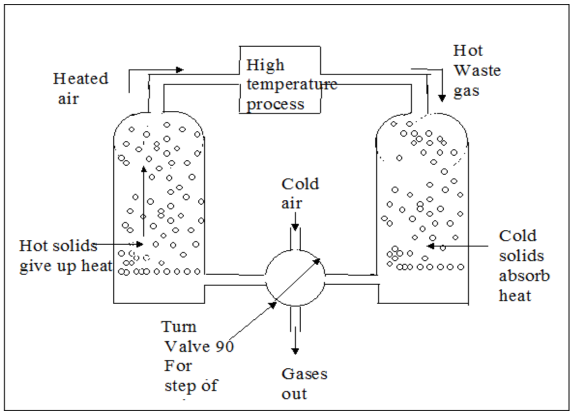

Fig.1. Heat transfer in a fixed bed of solids

The governing differential equations are derived by making an energy balance for the fluid and the packing material in an elemental control volume of length dz at position z within the fixed bed as shown in fig 2.

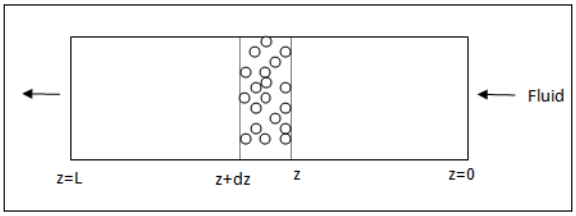

Fig. 2. Model of moving bed

The assumptions made to formulate the mathematical model are:

1) Fixed bed is one-dimensional, that is there is no variation along normal to the x-direction.
2) Axial conduction in solid phase is negligible.
3) Fluid velocity is constant along the bed.
4) There is no thermal gradient within the particle.
5) Thermal properties are independent of temperature.

The differential heat balance across dz for the gas phase is given by,

$$k_e \frac{\partial^2 T_g}{\partial z^2} - C_{pg}\rho_g v_z \frac{\partial T_g}{\partial z} + h_p a_s (T_s - T_g)/\varepsilon - \frac{4U}{D_b \varepsilon}(T_g - T_a) = C_{pg}\rho_g \frac{\partial T_g}{\partial t}$$

(1)

For the solid phase,

$$\rho_s C_{ps}(1-\varepsilon)\frac{\partial T_s}{\partial t} = h_p a_s (T_g - T_s) \quad (2)$$

Where,

a_s = specific surface per unit volume of packed bed

The initial and boundary conditions for the equations (1) and (2) are,

$$t = 0, \qquad 0 \le z \le L$$

$T_g = T_{ic}$, $T_s = T_{si}$(3)

Boundary conditions,

$$t > 0, \quad z = 0 \quad k_e \frac{dT_g}{dz} = v_g C_{pg}\rho_g \varepsilon (T_g - T_{gi})$$

$$z = L, \qquad \frac{\partial T_g}{\partial z} = 0 \qquad (4)$$

TABLE 1: PHYSICAL PARAMETERS FOR THE FIXED BED REGENERATOR

Packing material	Alumina
Bed length (m)	0.26
Bed diameter (m)	0.031
Diameter of the particle (m)	0.0034
Packing density (kg/m^3)	1840
Packing specific heat (J/kg.K)	1260
Thermal conductivity of Packing material (W/m.K)	3.114
Bed voidage	0.4
Gas flow rate (cc/s)	183.3
Superficial velocity (m/s)	0.2428
Particle Reynolds number (Re_p)	51.82
Peclet number	3156
h_p (W/m^2.K)	168.8
k_e (W/m.K)	0.7242
U (W/m^2.K)	0
Zone length (cm)	5.94

4 RESULTS AND DISCUSSION

By solving the both PDE's simultaneously with given initial and boundary conditions usingdata given in Table 1,the following results are obtained,

Temperature profiles along the bed length during heating the bed are shown in the fig 3 and 4. Figure 3 represents the results by finite difference method and fig 4. represents the results by wavelet method.

ISSN 2229-5518

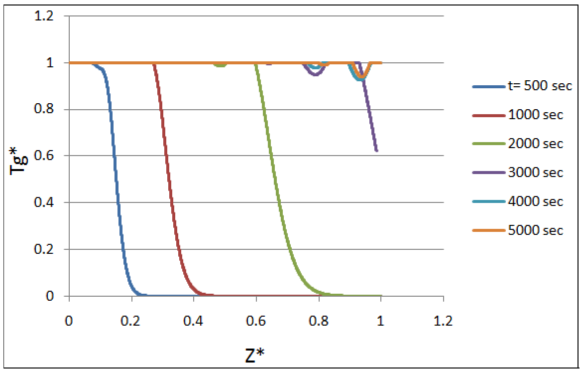

Fig.3. Gas temperature along the bed at different time intervals t (sec)

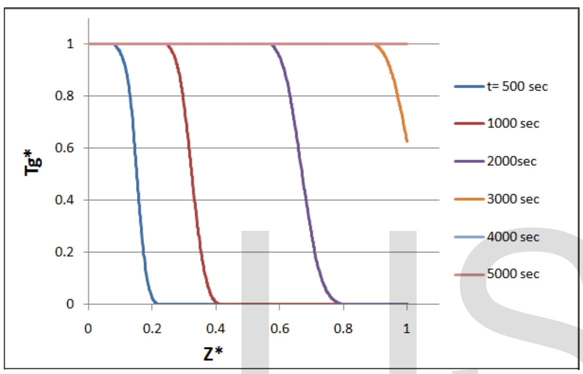

Fig. 4. Gas temperature along the bed at different time intervals t (sec)

Where,

$$T_g^* = \frac{T_g - T_{s0}}{T_{g0} - T_{s0}} , \qquad z^* = \frac{z}{L}$$

From the above figures it can be observed that there are number of errors in the temperature profiles by FDM in fig 3. These errors may be due to truncational as well as computational error. Whereas there is a smooth temperature profile in fig 4, obtained in case of wavelet method.

Breakthrough curve**s, t**he plot of outlet gas temperature with time is shown in the figs 5 and 6. Towards the end of the breakthrough curve it is observed that T_g^* is greater than 1 which is practically not possible as depicted in fig 5 and is due to numerical error arising from the FDM method used for solving the model PDE's.

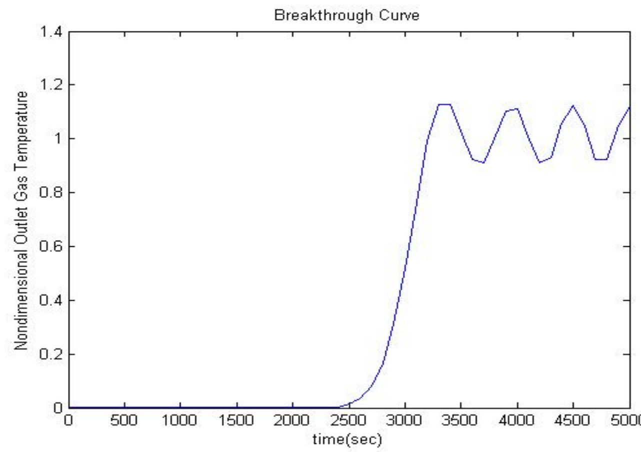

Fig.5. Breakthrough curve obtained using Finite Difference Method

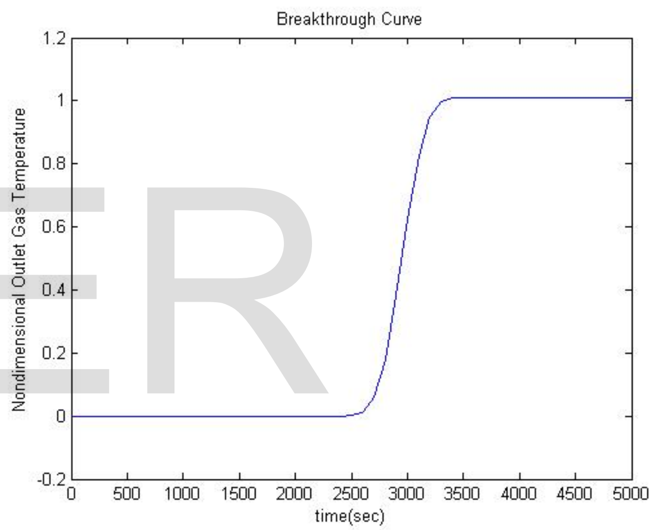

Fig.6. Breakthrough curve obtained using Wavelet

From the breakthrough curve it can be observed that there is a smooth temperature profile obtained by using wavelet while in case of FDM there are number of irregularities. The computational time in the case of FDM was 545 s , where as in case of wavelet the computational time was 155 s.

Wavelet based adaptive grid method involves the use of sparse grid. The minimum and maximum grid densities were 33 and 258for j=5 and j=10 respectively as observed in fig 7 and in fig 8. Wherever there are sharp changes in the temperature gradient , the grid density is increased automatically and where the gradient is smooth, the grid density is less.

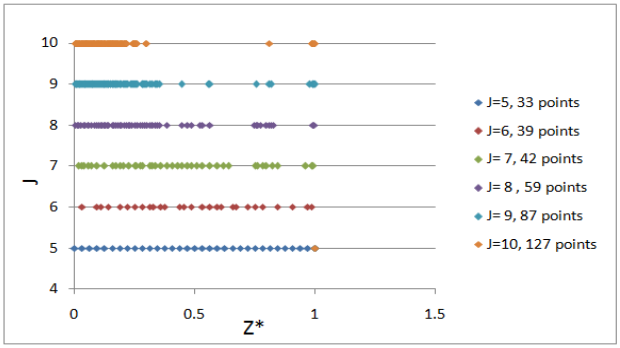

Fig. 7. Grid pattern at time t = 500 sec.

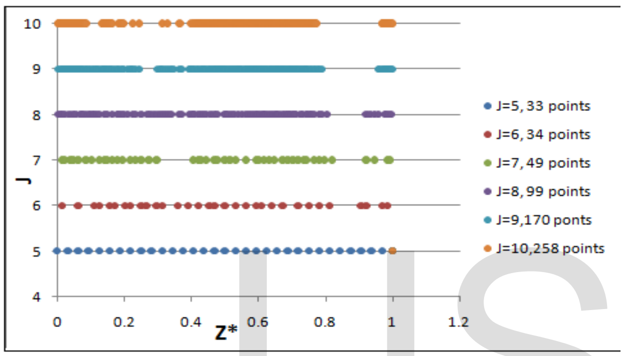

Fig.8. Grid pattern at time t = 2000 sec.

5 CONCLUSION

From this study, it can be said that with the help of wavelet based adaptive grid method, the grid can be adapted very easily and the adapted grid model gave the solution accurately. The time taken for the solution was very less because of the grid density being more only where it was required. The computational time in the case of **Wavelet**was very less as compare to FDM. The errors involved in the solution using wavelet were very less in comparison to FDM. So the advantage of wavelet based adaptive grid method is that it reduces the computational time as well as provides accurate results for solving the PDEs for fixed bed heat regenerator.

NOMENCLATURE

Symbol	
ϕ(x)	Scaling Function
ψ(x)	Mother Wavelet
ε	Threshold wavelet coefficient

REFERENCES

[1] Bhattacharyya, D. and D.C.T. Pei, "Heat Transfer in Fixed Bed Gas-Solid Systems" Chem. Eng. Sci., 30, 293 (1975)

[2] R. Williams and K. Amaratunga, Introduction to wavelets in engineering', IESL Technical Report No. 92-07, Intelligent Engineering Systems Laboratory, M.I.T., October 1992.

[3] Bertoluzza, S. (1996). A wavelet collocation method for the numerical solution of partial differential equations. Applied and Computational Harmonic Analysis 3, 1_/9.

[4] Lui, Y., Cameron, T.F., & Wang , Y.(2000) The wavelet_collocation method for transient problems with steep gradients. Chemical Engineering science , 55 ,1729-1734.

[5] Wavelet-Based Adaptive Grid Method for the Resolution of Nonlinear PDEs, AIChE Journal, Volume 48, Issue 4, April 2002, Pages 774-785 Paulo Cruz, Adelio Mendes and Fernão D. Magalhães.

[6] Design of long heat regenerators by use of the dispersion model, Octave Levenspiel, Oregon State University, Corvallis, OR 97331, U.S.A.

[7] Raghuvieer M.. Rao .Ajit s. Bopardikar, "Wavelet transforms,Introduction to theory and applications" Addison-Wesley

[8] Matlab wavelet toolbox users manual.